

Course Justification:

Mathematical Reasoning with Connections (MRWC) is a fourth year high school mathematics course designed to prepare students for the expectations and rigor of college mathematics courses. It reinforces and builds on mathematical topics and skills developed in Integrated Math (IM) 1-3 (or Algebra 1-2 and Geometry) and prepares students for mathematics courses required in either STEM and non-STEM majors.

Core Conceptual Understandings Acquired:

Based on the Common Core State Standards viewpoint that mathematics is a cohesive and connected body of work, the MRWC is structured to highlight conceptual connections in the more advanced study of topics leading to calculus. Emphasis is given to conceptual understanding and making connections between numerical, symbolic, verbal, and graphical representations, discussion and analysis of alternative representations and multiple perspectives for approaching and understanding. The distinctiveness of MRWC lies in its unique design and topic sequencing, and in the emphasis on instructional delivery that promotes exploratory and collaborative student engagement. MRWC seamlessly interweaves the CCSS Mathematical Practices throughout the curriculum and develops key Habits of Mind and a mathematical disposition required for mastering advanced, challenging college-level content knowledge.

Learning Skills Acquired:

MRWC uses a non-traditional instructional approach emphasizing collaboration and exploration through mathematical activities, problem posing, and the use of technology that will address diverse learning styles. Instruction is designed to challenge students to approach mathematics as sense-making through a focus on questioning and probing deeper. Teacher-led instruction and student explorations will focus on discovering

the conceptual basis for standard procedures. It will facilitate the development of students' ability to choose strategically among multiple solutions options, and to articulate the reasons for those decisions. Students will use informal and formal justifications to defend their understandings and critique the reasoning of others. Instruction will emphasize the use of and fluency in the full range of the language of mathematics. Content topics will be approached through six instructional modalities i.e. verbal, numeric, symbolic, graphical, geometric, and technological.

Social Skills Acquired:

Different forms of formative and summative assessments will be used. Students will demonstrate their ongoing conceptual understanding and procedural fluency through mathematical activities, small group discussions and explorations, personal reflection quick writes, in addition to worksheets and individual written assessments such as quizzes, tests, final summative exams. Students will also be assessed through group projects, oral and written presentations.

Measurable Goals:

The students will:

- A. Extend their work with real and complex numbers
- B. Represent complex numbers in the Cartesian plane and interpret operations on complex numbers as geometric transformations
- C. Represent complex numbers in polar and trigonometric form
- D. Prove trigonometric identities for compound angles to find powers and roots of complex numbers
- E. Analyze commonalities across families of functions that include algebraic functions such as absolute value, root, polynomial, rational, and

reciprocal, as well as transcendental functions such as exponential, logarithmic, and trigonometric

F. Extend basic trigonometric functions to reciprocal and inverse trigonometric functions

G. Determine basic ellipses and hyperbolas and use translated and rotated axes to graph all conics in non-standard positions

H. Graph advanced rational and piece-wise functions

I. Identify zeros, multiple roots, intercepts, symmetries, vertical/horizontal, and slant asymptotes, holes, and end-behavior of functions

J. Graph using parametric equations

K. Develop fluency and flexibility with both the algebraic and geometric meaning and interpretation of functional notation

L. Identify and determine formulas of functions given in tables, graphs, and real-world situations

M. Use underlying structure and the technique of u-substitution to simplify and solve advanced expressions, equations, and inequalities involving algebraic and trigonometric terms

N. Extend formulae to find distances in 3-dimensional space using algebraic and vector techniques, building on knowledge of distance as an application of the Pythagorean Theorem

O. Solve absolute value equations and inequalities by identify centers of intervals

P. Determine loci of curves and relate them to distances from foci in parabolas, ellipses, and hyperbolas

Q. Evaluate the effect of eccentricity as it relates to distance and the shapes of conics

R. Compare similarities among conics by identifying the number of parameters involved in describing the conic

S. Connect regression to distance of the residuals

T. Analyze rate of change with a secant line, and investigate the derivative as the limiting case of the slope of a secant

Alignment to CA Frameworks or Common Core Standards:

A. Reasoning with Numbers

1. Deepen conceptual understanding of the relationships between and the structures of various subsets of numbers that make up the Complex Number System. (N.CN.3)

2. Find conjugates of complex numbers and use them to divide complex numbers.(N.CN.3)

3. Find moduli of complex numbers and relate them to distance and absolute value.(N.CN.6)

4. Represent complex numbers in the Cartesian (Argand) plane and describe the algebraic operations on complex numbers in terms of geometric transformations of translation, dilation, and rotation. (N.CN.5)

5. Connect complex numbers to vectors in the plane, and relate modulus of a complex number to length of a vector. (N.CN.6, N.VM.1)

6. Relate scalar and vector multiplication to multiplication of complex numbers.(N.VM.4-5)

7. Represent complex numbers in polar and trigonometric forms and prove de Moivre's formula for multiplying and finding rational roots of complex numbers in trigonometric form. (N.VM.4-5)

8. Explore symmetries in the multiple roots of complex numbers and use the symmetries to explore infinite geometric sequences of complex numbers. (N.VM.3-5)

9. Prove that various subsets of the real and complex numbers are closed under different operations (including division, powers, and rooting).

(N.VM.5-6)

10. Identify irrational numbers as limiting values of infinite sequences, including nested radicals and continued fractions. (N.VM.5-6)

11. Prove the existence and magnitude of numbers of irrational and complex numbers through geometric construction and algebraic proof.

(N.VM.5-6)

B. Reasoning with Functions

1. Link patterns of real numbers to discrete functions, including arithmetic and geometric sequences and series. (F.IF.4-5)

2. Identify anomalies in the domain of continuous functions, including vertical asymptotes and removable points of discontinuity. (F.IF.4, F.IF.7d) 3.

Use numeric limits and algebraic procedures to identify whether a number that is excluded from the domain is a removable point of discontinuity or a vertical asymptote. (F.IF.4-5, F.IF.7d)

4. Use numeric limits to explore function behavior on either side of a vertical asymptote. (F.IF.4-5, F.IF.7d)

5. Identify key concepts related to functions including advanced study of domain and range, roots, symmetries and periodicity, positive/negative and increasing/decreasing. (F.IF.4-5)

6. Use algebraic factoring to predict function behavior based on multiplicity of roots and to find intervals on which functions are increasing/decreasing and positive/negative/constant. (F.IF.4, F.IF.7d)

7. Create functions given information about function features and behaviors. (F.IF.4, F.IF.7d, F.BF.3)

- 8. Create functions in two or more variables that represent relationships between quantities expressed in verbal, numeric or graphical form. (F.IF.4-5, F.BF.3)**
- 9. Use numeric limits and algebraic procedures to identify and describe the end behavior of a function, including limits at infinity, horizontal and slant asymptotes. (F.IF.4-5)**
- 10. Study the graphs and features of reciprocal and inverse functions. (F.IF.7e, F.BF.3, F.BF.4b-d)**
- 11. Relate features of reciprocal and inverse functions to understand trigonometric functions of cosecant, secant, cotangent and the inverse trigonometric functions. (F.BF.4b-d)**
- 12. Use completing the square techniques to graph ellipses and hyperbolas in standard and non-standard positions. (G.GPE.3.1, CA)**
- 13. Use trigonometric techniques to draw rotated conics. (G.GPE.3)**
- 14. Study parametric forms of equations and relate them to transformations. (F.BF.10, CA)**
- 15. Make connections between geometrical transformations (such as translation, rotation, reflections, dilations and stretches of graphs) and the algebraic process of function composition. (F.BF.3-4)**
- 16. Expand composition to include composition of three or more functions. (F.BF.4)**
- 17. Create new function graphs by composing functions given in graphical or tabular representations. (F.IF.4, F.IF. 7, F.BF.3-4)**
- 18. Decompose complicated functions into component functions, both graphically and algebraically. (F.IF.4, F.IF. 7, F.BF.3-4)**
- 19. Study basic properties of matrices and vectors. (N.VM.4-8)**
- 20. Use vectors and matrices as a means to represent function transformations. (N.VM.9-12)**

21. Use parametric equations to graph advanced functions. (F.IF.10, CA) 22. Interpret function notation and function composition graphically, verbally, numerically and algebraically. (A.SSE.1-2, F.IF.4, F.IF.7)
23. Use function notation to prove features of functions such as odd/even, increasing/decreasing, the existence of symmetry lines in parabolas and other conics. (F.BF.3, F.IF.10, CA)

C. Reasoning with Equivalences

1. Prove/disprove identities among equivalent and non-equivalent expressions involving polynomial, rational, root, exponential, and logarithmic terms. (A.SSE.1-2)
2. Prove/disprove trigonometric identities involving co-functions, compound/double/half angle formula. (F.TF.6-9, F.TF.10, CA)
3. Use similarity of triangles to develop sine and cosine and area formula for solving non-right triangles. (G.SRT.9-11)
4. Develop fluency and flexibility in manipulating complicated forms of composite expressions and equations (including advanced factoring) by identifying and strategically using the idea of underlying structure. (A.SSE.1-2)
5. Use advanced factoring techniques on expressions and equations involving binomials with rational exponents and terms with logarithmic and trigonometric exponents. (A.SSE.1-2)
6. Solve advanced composite equations and inequalities involving polynomial, rational, root, absolute value, trigonometric, exponential and logarithmic expressions by identifying and strategically using underlying structure and alternative representations. (A.SSE.1-2, A.APR.6-7)
7. Identify and solve trigonometric equations and inequalities that have underlying structure of polynomial form using algebraic and graphical techniques. (F.TF.6-7, A.CED.1-3)

8. Represent and solve systems of linear equations using matrices and a vector variable and use matrix inversion to solve. (A.CED.2-3, A.REI.8-9)
9. Solve advanced systems of non-linear equations and inequalities involving roots, absolute value, exponentials, logarithms and trigonometric terms. (A.CED.2-3)
10. Create equations in two or more variables to represent relationships between quantities. (A.CED.1-2)
11. Explore the geometry of polygons, curves, perimeter and area through equivalences such as similarity and congruence and transformations that preserve perimeter and/or area. (N.VM.11-12)
12. Study parallelism as an equivalence relation through two- and three-dimensional vectors. (N.VM.11-12)

Other Content Taught:

D. Distances

1. Extend the concept of distance as absolute value to find distances and midpoints between points in 3-dimensional space. (N.CN.6, N.VM.12)
2. Use the concept of loci to explore conics and other curves in algebraic and polar form. (G.GPE.3)
3. Use real world data sets to connect the least square method of linear regression to the measurement of residuals as distances. (S.ID.6)
4. Extend the concept of distance to study slope, rate of change and secant lines. (S.ID.7, F.IF.6, CA Math Framework: Calculus 4.1-4.2)
5. Explore the slope of a tangent line to a curve as the limiting case of the slope of a secant, and develop the concept of a derivative as a point. (S.ID.7, CA Math Framework: Calculus 4.1-4.2)

21st Century Skills for College and Career Acquired:

From their high school mathematics courses, students should have gained certain approaches, perspectives, and dispositions:

- 1. A view that mathematics makes sense - Students should perceive mathematics as a way of understanding, not as a sequence of algorithms to be memorized and applied.**
- 2. An ease in using their mathematical knowledge to solve unfamiliar problems in both concrete and abstract situations - Students should be able to find patterns, make conjectures, and test those conjectures; they should recognize that abstraction and generalization are important sources of the power of mathematics; they should understand that mathematical structures are useful as representations of phenomena in the physical world; they should consistently verify that their solutions to problems are reasonable.**
- 3. A willingness to work on mathematical problems requiring time and thought, problems that are not solved by merely mimicking examples that have already been seen - Students should have enough genuine success in solving such problems to be confident, and thus to be tenacious, in their approach to new ones.**
- 4. A readiness to discuss the mathematical ideas involved in a problem with other students and to write clearly and coherently about mathematical topics - Students should be able to communicate their understanding of mathematics with peers and teachers using both formal and natural languages correctly and effectively.**
- 5. An acceptance of responsibility for their own learning - Students should realize that their minds are their most important mathematical resource, and that teachers and other students can help them to learn but cannot learn from them.**
- 6. The understanding that assertions require justification based on persuasive arguments, and an ability to supply appropriate justifications -**

Students should habitually ask "Why?" and should have a familiarity with reasoning at a variety of levels of formality, ranging from concrete examples through informal arguments using words and pictures to precise structured presentations of convincing arguments.

7. While proficiency in the use of technology is not a substitute for mathematical competency, students should be familiar with and confident in the use of computational devices and software to manage and display data, to explore functions, and to formulate and investigate mathematical conjectures.

8. A perception of mathematics as a unified field of study - Students should see interconnections among various areas of mathematics, which are often perceived as distinct.

Course Outline Requirements

Overview:

Mathematical Reasoning with Connections (MRWC) is designed as a fourth-year, advanced mathematics course following Integrated Math I - 3 sequence of courses. The MRWC course is designed to prepare students for college level mathematics including pre-calculus, calculus, and other quantitative reasoning courses; the course satisfies the "c" Advanced Mathematics requirement for UC "a-g" criteria. Based on the Common Core State Standards, MRWC is structured to highlight conceptual connections in the more advanced study of topics leading to calculus. Emphasis is given to conceptual understanding and making connections between numerical, symbolic, verbal, and graphical representations, discussion and analysis of alternative representations and multiple perspectives for approaching and understanding. The distinctiveness of MRWC lies in its unique design and topic sequencing, and in the emphasis on instructional delivery that

promotes exploratory and collaborative student engagement. MRWC seamlessly interweaves the CCSS Mathematical Practices throughout the curriculum and develops key Habits of Mind and a mathematical disposition required for mastering advanced, challenging college-level content knowledge.

Course Goals:

The students will:

- A. Extend their work with real and complex numbers**
- B. Represent complex numbers in the Cartesian plane and interpret operations on complex numbers as geometric transformations**
- C. Represent complex numbers in polar and trigonometric form**
- D. Prove trigonometric identities for compound angles to find powers and roots of complex numbers**
- E. Analyze commonalities across families of functions that include algebraic functions such as absolute value, root, polynomial, rational, and reciprocal, as well as transcendental functions such as exponential, logarithmic, and trigonometric**
- F. Extend basic trigonometric functions to reciprocal and inverse trigonometric functions**
- G. Determine basic ellipses and hyperbolas and use translated and rotated axes to graph all conics in non-standard positions**
- H. Graph advanced rational and piece-wise functions**
- I. Identify zeros, multiple roots, intercepts, symmetries, vertical/horizontal, and slant asymptotes, holes, and end-behavior of functions**
- J. Graph using parametric equations**
- K. Develop fluency and flexibility with both the algebraic and geometric meaning and interpretation of functional notation**

- L. Identify and determine formulas of functions given in tables, graphs, and real-world situations**
- M. Use underlying structure and the technique of u-substitution to simplify and solve advanced expressions, equations, and inequalities involving algebraic and trigonometric terms**
- N. Extend formulae to find distances in 3-dimensional space using algebraic and vector techniques, building on knowledge of distance as an application of the Pythagorean Theorem**
- O. Solve absolute value equations and inequalities by identify centers of intervals**
- P. Determine loci of curves and relate them to distances from foci in parabolas, ellipses, and hyperbolas**
- Q. Evaluate the effect of eccentricity as it relates to distance and the shapes of conics**
- R. Compare similarities among conics by identifying the number of parameters involved in describing the conic**
- S. Connect regression to distance of the residuals**
- T. Analyze rate of change with a secant line, and investigate the derivative as the limiting case of the slope of a secant**

Expected Outcomes:

Students will

- 1. deepen their conceptual understanding of the relationships between and the structures of various subsets of numbers that make up the complex number system.**
- 2. find conjugates of complex numbers and use them to divide complex numbers.**
- 3. find moduli of complex numbers and relate them to distance and absolute value.**

4. represent complex numbers in the Cartesian (argand) plane and describe the algebraic operations on complex numbers in terms of geometric transformations of translation, dilation, and rotation.
5. connect complex numbers to vectors in the plane, and relate modulus of a complex number to length of a vector.
6. relate scalar and vector multiplication to multiplication of complex numbers.
7. represent complex numbers in polar and trigonometric forms and prove de Moivre's formula for multiplying and finding rational roots of complex numbers in trigonometric form.
8. explore symmetries in the multiple roots of complex numbers and use the symmetries to explore infinite geometric sequences of complex numbers.
9. prove that various subsets of the real and complex numbers are closed under different operations (including division, powers, and rooting).
10. identify irrational numbers as limiting values of infinite sequences, including nested radicals and continued fractions.
11. prove the existence and magnitude of numbers of irrational and complex numbers through geometric construction and algebraic proof.

Prerequisites:

C or better in Integrated 1-3 (or Algebra 1-2 and Geometry)

Co-requisites:

Recommended for MRWC:

SBAC = 2

EAP Conditional

SBAC = 3

EAP Ready SBAC = 4 or calculus-bound students who want to build a stronger foundation

Course Content:

Theme 1: Reasoning with Numbers

Students will explore the development of the Number System up through complex numbers. The mathematical idea of closure will provide a logical structure for the construction, expansion, and organization of the sets of numbers. Students will gain a deeper, more conceptual interpretation of each major subset (Natural, Integers, Rationals, Irrationals, Real, and Complex) of the Number System by considering three questions: What evidence is there that these numbers exist and are needed for subsequent understanding and production in mathematics? If these numbers exist, where are they positioned on the real number line or in the complex plane? How do the standard operations change in meaning and properties as the set of numbers is expanded? In answering these questions, students will explore the link between numerical symbols, algebraic representations, geometric construction, transformations, and concepts of limits as tools for understanding numbers and their behaviors and the rules that govern their use.

Students will

- 1. deepen their conceptual understanding of the relationships between and the structures of various subsets of numbers that make up the complex number system.**
- 2. find conjugates of complex numbers and use them to divide complex numbers.**

3. find moduli of complex numbers and relate them to distance and absolute value.
4. represent complex numbers in the Cartesian (argand) plane and describe the algebraic operations on complex numbers in terms of geometric transformations of translation, dilation, and rotation.
5. connect complex numbers to vectors in the plane, and relate modulus of a complex number to length of a vector.
6. relate scalar and vector multiplication to multiplication of complex numbers.
7. represent complex numbers in polar and trigonometric forms and prove de Moivre's formula for multiplying and finding rational roots of complex numbers in trigonometric form.
8. explore symmetries in the multiple roots of complex numbers and use the symmetries to explore infinite geometric sequences of complex numbers.
9. prove that various subsets of the real and complex numbers are closed under different operations (including division, powers, and rooting).
10. identify irrational numbers as limiting values of infinite sequences, including nested radicals and continued fractions.
11. prove the existence and magnitude of numbers of irrational and complex numbers through geometric construction and algebraic proof.

Theme 2: Reasoning with Functions.

Students will explore commonalities across families of functions that include algebraic functions such as absolute value, root, polynomial, rational, and reciprocal, as well as transcendental functions such as exponential, logarithmic, and trigonometric. Students will develop fluency and flexibility with both the algebraic and geometric meaning and

interpretation of functional notation. They will identify and find formulas of functions given in tables, graphs, and real world situations.

Students will

- 1. link patterns of real numbers to discrete functions, including arithmetic and geometric sequences and series.**
- 2. identify anomalies in the domain of continuous functions, including vertical asymptotes and removable points of discontinuity.**
- 3. use numeric limits and algebraic procedures to identify whether a number that is excluded from the domain is a removable point of discontinuity or a vertical asymptote.**
- 4. use numeric limits to explore function behavior on either side of a vertical asymptote.**
- 5. study key concepts related to functions including advanced study of domain and range, roots, symmetries and periodicity, positive/negative and increasing/decreasing.**
- 6. use algebraic factoring to predict function behavior based on multiplicity of roots and to find intervals on which functions are increasing/decreasing and positive/negative/constant.**
- 7. create functions given information about function features and behaviors.**
- 8. create functions in two or more variables that represent relationships between quantities expressed in verbal, numeric, or graphical form.**
- 9. use numeric limits and algebraic procedures to identify and describe the end behavior of a function, including limits at infinity, horizontal and slant asymptotes.**
- 10. study the graphs and features of reciprocal and inverse functions.**

11. relate features of reciprocal and inverse functions to understand trigonometric functions of cosecant, secant, cotangent, and the inverse trigonometric functions.
12. use completing the square techniques to graph ellipses and hyperbolas in standard and non-standard positions.
13. use trigonometric techniques to draw rotated conics.
14. study parametric forms of equations and relate them to transformations.
15. make connections between geometrical transformations (such as translation, rotation, reflections, dilations and stretches of graphs) and the algebraic process of function composition.
16. expand composition to include composition of three or more functions.
17. create new function graphs by composing functions given in graphical or tabular representations.
18. decompose complicated functions into component functions, both graphically and algebraically.
19. study basic properties of matrices and vectors.
20. use vectors and matrices as a means to represent function transformations.
21. use parametric equations to graph advanced functions.
22. interpret function notation and function composition graphically, verbally, numerically and algebraically.
23. use function notation to prove features of functions such as odd/even, increasing/decreasing, the existence of symmetry lines in parabolas and other conics.

Theme 3: Reasoning with Identities, Equations, and Inequalities

Students will explore mathematical properties and characteristics of basic algebraic and geometric entities in order to develop generalizations that can be applied to more complex situations. The mathematical idea of equivalence will provide the logical structure for manipulating expressions, solving equations and inequalities, and studying geometrical figures. Students will use underlying structure and the technique of u-substitution to simplify and solve advanced expressions, equations, and inequalities involving algebraic and trigonometric terms.

Students will

- 1. prove/disprove identities among equivalent and non-equivalent expressions involving polynomial, rational, root, exponential, and logarithmic terms.**
- 2. prove/disprove trigonometric identities involving co-functions, compound/double/half angle formula.**
- 3. use similarity of triangles to develop sine and cosine and area formula for solving non-right triangles.**
- 4. develop fluency and flexibility in manipulating complicated forms of composite expressions and equations (including advanced factoring) by identifying and strategically using the idea of underlying structure.**
- 5. use advanced factoring techniques on expressions and equations involving binomials with rational exponents and terms with logarithmic and trigonometric exponents.**
- 6. solve advanced composite equations, inequalities, involving polynomial, rational, root, absolute value, trigonometric, exponential, and logarithmic expressions by identifying and strategically using underlying structure and alternative representations.**

7. identify and solve trigonometric equations and inequalities that have underlying structure of polynomial form using algebraic and graphical techniques.
8. represent and solve systems of linear equations using matrices and a vector variable and use matrix inversion to solve.
9. solve advanced systems of non-linear equations and inequalities involving roots, absolute value, exponentials, logarithms, and trigonometric terms.
10. create equations in two or more variables to represent relationships between quantities
11. explore the geometry of polygons, curves, perimeter and area through equivalences such as similarity and congruence and transformations that preserve perimeter and/or area.
12. study parallelism as an equivalence relation through two- and three-dimensional vectors.

Theme 4: Reasoning with Distance.

Building on knowledge of distance as an application of the Pythagorean Theorem, students will extend formulae to find distances in 3-dimensional space using algebraic and vector techniques. They will solve absolute value equations and inequalities by identify centers of intervals. Students will explore loci of curves and relate them to distances from foci in parabolas, ellipses, and hyperbolas. They will explore the effect of eccentricity as it relates to distance and the shapes of conics. They will identify similarities among conics by identifying the number of parameters involved in describing the conic. Students will relate regression to distance of the residuals. Students will link rate of change with a secant line, and investigate the derivative as the limiting case of the slope of a secant.

Students will

- 1. extend the concept of distance as absolute value to find distances and midpoints between points in 3-dimensional space.**
- 2. find midpoints**
- 3. use the concept of loci to explore conics and other curves in algebraic and polar form.**
- 4. use real world data sets to connect the least square method of linear regression to the measurement of residuals as distances.**
- 5. extend the concept of distance to study slope, rate of change, and secant lines.**
- 6. explore the slope of a tangent line to a curve as the limiting case of the slope of a secant, and develop the concept of a derivative as a point.**

Course Materials:

MRWC provides

- Teacher Binder which provides content discussion, students activities and exercises, and pedagogical suggestions for delivering the curriculum**
- Classroom set of all mathematical puzzle games**
- PDFs of student activity worksheets to be copied by the district**

Course Fiscal Impact:

Funding Sources:

Texts:

Training:

Lab Supplies:

Durable Equipment:

Study Trips:

Cour